ON THE UNSTEADY CREEP OF PLATES AND SHELLS WITH SMALL DISPLACEMENTS

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We assume that the displacements are small and that the total deformations are the sum of creep deformations and instantaneous elasto-plastic deformations. "Instantaneous" means that during loading and unloading neither creep nor relaxation can be noticed.

1. Let us denote by **U** the total displacement vector, by **u** the elasto-plastic displacement vector and by **v** the creep displacement vector. Thus **U** = **u** + **v**. For the total deformation ε_{ik}, the creep deformation p_{ik}, and the elasto-plastic deformation q_{ik}, the following relations exist

$$2\varepsilon_{ik} = \nabla_i U_k + \nabla_k U_i, \qquad 2p_{ik} = \nabla_i v_k + \nabla_k v_i, \qquad 2q_{ik} = \nabla_i u_k + \nabla_k u_i$$
$$\varepsilon_{ik} = q_{ik} + p_{ik}, \qquad U_k = \mathbf{U}\cdot\mathbf{r}_k, \qquad u_k = \mathbf{u}\cdot\mathbf{r}_k, \qquad v_k = \mathbf{v}\cdot\mathbf{r}_k \tag{1.1}$$

Here ε_{ik}, p_{ik} and q_{ik} are the covariant components of the corresponding strain tensors. The nonlinear terms in (1.1) are omitted by virtue of the assumption of small displacements. Further, $\nabla_i(\ldots)$ represents the covariant derivative of the metric with the metric tensor $g_{ik} = \mathbf{r}_i \times \mathbf{r}_k$; $\mathbf{r}_k$ are the coordinate vectors in the curvilinear system x^i (i = 1, 2, 3) introduced in the body of volume V. For the velocity of creep deformation ξ_{ik} we have

$$2\xi_{ik} = \nabla_i \dot{v}_k + \nabla_k \dot{v}_i, \qquad \dot{v}_i = \frac{dv_i}{dt}, \qquad \xi_{ik} = \frac{dp_{ik}}{dt} \tag{1.2}$$

The creep relations are taken from the theory of flow [1]

$$\xi_{ik} = \frac{3}{2} g(\Gamma, T)(\sigma_{ik} - g_{ik}\sigma), \qquad \Gamma = \int_0^t H dt, \qquad T^2 = \frac{3}{2}(\sigma^{ik}\sigma_{ik} - 3\sigma^2) \tag{1.3}$$
$$H^2 = {}^2/_3\, \xi_{ik}\xi^{ik}, \quad 3\sigma = \sigma^{ik} g_{ik} = \sigma_i^i, \qquad \xi = \xi_{ik} g^{ik} = \xi_i^{\ i} = 0$$

Here σ^{ik} are the contravariant components of the stress tensor. The function $g(\Gamma, T)$ is obtained by experiment and is taken in the form [1,2]

$$g(\Gamma, T) = A\Gamma^{-d}T^{n-1} \qquad (A, d, n = \text{const}) \tag{1.4}$$

From the relation (1.3) it follows that

$$H = g(\Gamma, T)\, T \tag{1.5}$$

For the solution of problems it is convenient to use the following statement.

Among all states allowed by the kinematical constraints (1.2) in the body and the kinematical relations on the surface of the body, only those occur which assign a stationary (minimum) value to the functional

$$J = \iiint\limits_V A^{-\mu}\Gamma^{d\mu}\frac{H^{1+\mu}}{1+\mu}\,dV - \iiint\limits_V \mathbf{Q}\cdot\dot{\mathbf{v}}dV - \iint\limits_S \mathbf{P}\cdot\dot{\mathbf{v}}dS \qquad \left(\mu = \frac{1}{n}\right) \tag{1.6}$$

Denote by δN the variation of the power of the external loads **P** and the body forces **Q** in the variations of the creep displacement rates

$$\delta N = \iiint\limits_V \mathbf{Q}\cdot\delta\dot{\mathbf{v}}dV + \iint\limits_S \mathbf{P}\cdot\delta\dot{\mathbf{v}}dS \qquad \left(\dot{\mathbf{v}} = \frac{d\mathbf{v}}{dt}\right) \tag{1.7}$$

The power of internal stresses in the variation of creep displacement rates is

$$\delta M = \iiint\limits_V \sigma^{ik}\delta\xi_{ik}dV \tag{1.8}$$

Since $g = A^{\mu}\Gamma^{-d\mu}H^{1-\mu}$, $\delta\Gamma = 0$, $\xi^i = 0$, $\delta P = \delta Q = 0$, then

$$\delta M = \delta\iiint\limits_V \frac{A^{-\mu}\Gamma^{d\mu}H^{1+\mu}}{1+\mu}\,dV, \qquad \delta J = \delta(M - N)$$

We shall show that for the actual states

$$\delta(M - N) = 0 \tag{1.9}$$

Here only variations in the velocity and the displacement of creep deformation are admitted. Since $\sigma^{ik}\delta\xi_{ik} = \sigma^{ik}\mathbf{r}_k \times \delta\partial\mathbf{v}/\partial x^i$ then

$$\delta(M - N) = \iiint\limits_V \sigma^{ik}\mathbf{r}_k\cdot\delta\frac{\partial\dot{\mathbf{v}}}{\partial x^i}\,dV - \iiint\limits_V \mathbf{Q}\cdot\delta\dot{\mathbf{v}}dV - \iint\limits_S \mathbf{P}\cdot\delta\dot{\mathbf{v}}dS \tag{1.10}$$

Applying to the first term the Ostrogradsky-Gauss theorem, we obtain

$$\delta(M-N)=-\iiint_V\{\nabla_i\sigma^{ik}+Q^k\}\delta\dot v_k dV-\iint_S\{\sigma^{ik}n_i+P^k\}\delta\dot v_k dS \qquad (1.11)$$

Consequently, if all static conditions are satisfied, and in addition geometric and kinematic relations are not violated, then $\delta(M-N)=0$. Conversely, if all geometric relations are satisfied, then by virtue of the independency of δv_k in and on the body, the relations $\delta(M-N)=0$ yield the equation of equilibrium. Since the quadratic form

$$\phi=\frac{\partial^2E}{\partial\xi_{ik}\partial\xi_{jn}}\overset{\circ}{\xi}_{ik}\overset{\circ}{\xi}_{jn}=A^{-\mu}\Gamma^{d\mu}\frac{1}{1+\mu}\frac{\partial^2H^{1+\mu}}{\partial\xi_{ik}\partial\xi_{jn}}\overset{\circ}{\xi}_{ik}\overset{\circ}{\xi}_{jn}$$

is positive-definite at $\xi^d_{ik}\neq 0$ (for proof of this statement the method of L.M. Kachanov [3, p.109] can be employed), then the function

$$E=A^{-\mu}\Gamma^{d\mu}H^{1+\mu}/(1+\mu)$$

is convex in the sense of [5], namely

$$E(\xi^*_{jn})-E(\xi_{jn})\geqslant(\xi^*_{ik}-\xi_{ik})\frac{\partial E}{\partial\xi_{ik}}$$

For the satisfaction of the last condition it is necessary and sufficient that the functional J reaches a minimum with respect to actual velocity [5].

2. In order to write the functional J for thin plates and shells, we assume that the stresses σ^{33} are small compared with the stresses $\sigma^{\alpha\beta}\sqrt{a_{\alpha\alpha}}\sqrt{a_{\beta\beta}}$ (no sum with respect to α and β), and also we assume the absence of shearing ε_{13}, ε_{23}. Here the coordinate $x^3=z$ is measured along the normal $\mathbf{m}$ of the middle surface S_0, to which the curvilinear system x^α ($\alpha=1,2$) is connected with the coordinate vectors ρ_α. The normal unit vector $\mathbf{m}$ on the surface S_0 is obtained from the relation

$$\mathbf{m}c_{\alpha\beta}=\rho_\alpha\times\rho_\beta,\quad c_{12}=-c_{21}=\sqrt{a},\quad c_{11}=c_{12}=0,\quad a=\det(a_{\alpha\beta}),\quad a_{\alpha\beta}=\rho_\alpha\cdot\rho_\beta \qquad (2.1)$$

The displacement vector of creep can be represented as

$$\mathbf{v}=(v_\alpha-z\omega_\alpha)\rho^\alpha+v_3\mathbf{m} \qquad (2.2)$$

For the deformation and for the velocity of deformation of creep one obtains

$$\begin{gathered}2p_{\alpha\beta}=\nabla_\alpha v_\beta+\nabla_\beta v_\alpha-2b_{\alpha\beta}v_3-z(\nabla_\alpha\omega_\beta+\nabla_\beta\omega_\alpha)\\2\xi_{\alpha\beta}=\nabla_\alpha\dot v_\beta+\nabla_\beta\dot v_\alpha-2b_{\alpha\beta}\dot v_3-z(\nabla_\alpha\dot\omega_\beta+\nabla_\beta\dot\omega_\alpha)\\\omega_\alpha=\nabla_\alpha v_3+b_\alpha^{\ \lambda}v_\lambda\qquad(\dot\omega_\alpha=d\omega_\alpha/dt)\end{gathered} \qquad (2.3)$$

Here $\nabla_\alpha(\ldots)$ denotes the covariant derivative with respect to the metric of the surface S, determined by metric tensor $a_{\alpha\beta}$. Moreover, $b_{\alpha\beta}$ denotes the covariant components of the second base tensor of the surface S_0.

From (1.3) and using the assumption that σ^{33} is small, taking into account $\xi_{33} = -\xi^\alpha_a$, we have

$$\sigma^{\alpha\beta} = \frac{2}{3g}\{\xi^{\alpha\beta} + a^{\alpha\beta}\xi_{\lambda\cdot}^{\lambda}\} \tag{2.4}$$

By virtue of these assumptions, the functional (1.6) for thin plates and shells can be written in the form

$$J_* = \iint\limits_S \int\limits_{-h}^{h} A^{-\mu}\Gamma_*^{d\mu}\frac{H_*^{1+\mu}}{1+\mu}\,dzdS_0 \iint\limits_{S_+} \mathbf{P}_+\{(\dot v_\alpha - h\dot w_\alpha)\rho^\alpha + \dot v_3\mathbf{m}\}\,dS_0 -$$
$$-\iint\limits_{S_-} \mathbf{P}_-\{(\dot v_\alpha + h\dot w_\alpha)\rho^\alpha + \dot v_3\mathbf{m}\}\,dS_0 - \int\limits_L\int\limits_{-h}^{h}\mathbf{P}_L\{(\dot v_\alpha - z\dot w_\alpha)\rho^\alpha + \dot v_3\mathbf{m}\}\,dzdL \tag{2.5}$$

where

$$\Gamma^* = \int\limits_0^t H_* dt, \qquad H_*^2 = \frac{2}{3}\{\xi_{\alpha\beta}\xi^{\alpha\beta} + \xi_{\lambda\cdot}^{\lambda}\xi_{\gamma\cdot}^{\gamma}\} \tag{2.6}$$

and $2h$ is the thickness of the shell, L is boundary of the middle surface, $\mathbf{P}_L$ is the loading vector acting on the shell surface; $\mathbf{P}_+$ and $\mathbf{P}_-$ denotes the load on the surfaces $S_+(z = h)$ and $S_-(z = -h)$, respectively.

3. We will use the functional (2.5) in order to solve the problem of unsteady creep of a circular plate clamped along the edges, under uniform transverse loading of intensity q.

We are looking for an approximate form, which results from the corresponding problem of the linear theory of an elastic plate, namely

$$v_3 = f(1-\eta^2)^2, \qquad v_1 = v_2 = 0, \qquad f = f(t) \tag{3.1}$$

In the case considered we have

$$a_{11} = r^2,\ a_{22} = r^2\eta^2, \qquad b_{\alpha\beta} = 0, \qquad \Gamma_2{}^2{}_1 = \frac{1}{\eta}, \qquad \Gamma_2{}^1{}_2 = -\eta \tag{3.2}$$

Here r is the radius of the plate, $\Gamma^\lambda_{\alpha\beta}$ is the Christoffel symbol of the second kind, $0 < \eta < 1$, and the distance between any arbitrary point of the plate and a point on its axis is equal to $r\eta$.

Substituting v_i from (3.1) into (2.3) and using (3.2), one obtains for J_*

$$J_* = \frac{2\pi r^2 h}{1+\mu} A^{-\mu} f^{d\mu} \dot{f}^{1+\mu} \int_0^1 \eta d\eta \int_{-1}^{1} \left(\frac{8h^2}{\sqrt{3}r^2}\zeta\varkappa'\right)^{1+\mu(1+d)} d\zeta - 2\pi r^2 q \dot{f} \int_0^1 (1-\eta^2)^2 \eta d\eta \quad (3.3)$$

where

$$\varkappa^2 = 3 - 12\eta^2 + 13\eta^4, \qquad \zeta = z/h, \qquad \dot{f} = df/dt \quad (3.4)$$

Since for the actual state $\delta J_* = 0$, then the time-function $f(t)$ is determined by

$$f^{d\mu} \dot{f}^{\mu} = A^{\mu} q (2n + d + 1) \left\{12n \int_0^1 \left(\frac{8h^2}{\sqrt{3}r^2}\varkappa\right)^{1+\mu(1+d)} \eta d\eta\right\}^{-1} \quad (3.5)$$

which for $dq/dt = 0$ yields

$$f^{d+1} = At(d+1) q^n (2n + d + 1)^n \left\{12n \int_0^1 \left(\frac{8h^2}{\sqrt{3}r^2}\varkappa\right)^{1+\mu(1+d)} \eta d\eta\right\}^{-n} \quad (3.6)$$

Thus, the total displacement of the plate is given by

$$U_3 = v_3 + u_3, \qquad U_1 = U_2 = 0$$

where u_3 is the elasto-plastic solution resulting from the geometrically linear theory of thin plates.

4. The method outlined above in Sections 1 and 2 gives the possibility of the determination of the displacement and deformation under unsteady creep conditions. Determination of the stress according to the relation (1.3) and (2.4), or according to the relations of the elasto-plastic deformation, cannot give the actual distribution, since from those relations one obtains different stress distributions. For the solution of this problem it is convenient to use the method suggested in [3] based on the variational principle of admissible variation of the state of stress.

Let us use the fact that in our case this assumption is met (an analogous assumption in the absence of instantaneous plastic deformation was proved in [6]).

Among all states of stress, which do not violate the static conditions inside the body and on the surface, only those actually occur which assign a stationary value to the functional

$$K = \iiint_V \left\{\frac{A}{n+1}\Gamma^{-d} T^{n+1} + \frac{d}{dt} R\right\} dV - \iiint_V \dot{\mathbf{v}} \cdot \mathbf{Q} dV - \iint_S \dot{\mathbf{v}} \cdot \mathbf{P} dS \quad (4.1)$$

Here by R is denoted the density of the complementary work in elasto-plastic deformations [4]

$$R = U + \int_0^\tau \theta(\tau)\,\tau d\tau \tag{4.2}$$

where U is energy of volume deformation and

$$\gamma = \theta(\tau)\,\tau, \qquad \tau = \frac{1}{\sqrt{3}}T, \qquad \gamma^2 = 2\left(q_{ik}q^{ik} - \frac{1}{3}q_{i\cdot}^{i}q_{k\cdot}^{k}\right) \tag{4.3}$$

If the plastic deformations are absent, we have

$$R = U + \frac{1}{2G}\tau^2 = \Pi$$

where Π is the energy density of elastic deformation.

In the functional (4.1) the variation is permitted only in statical characteristics. Note that the variation of the stresses is independent of time t, i.e.

$$\frac{d}{dt}\delta\sigma^{ik} = 0, \qquad \frac{d}{dt}\delta \mathbf{Q} = 0, \qquad \frac{d}{dt}\delta \mathbf{P} = 0 \tag{4.4}$$

In order to prove the given statement we construct the expression for the power of the variations of internal stresses in actual velocities of the total deformation and the power of the variation of external forces in actual total displacement velocities

$$\delta K_1 = \iiint\limits_V \dot{\varepsilon}_{ik}\delta\sigma^{ik}dV - \iiint\limits_V \dot{\mathbf{U}}\cdot\delta\mathbf{Q}dV - \iint\limits_S \dot{\mathbf{U}}\cdot\delta\mathbf{P}dS \tag{4.5}$$

since

$$\dot{\varepsilon}_{ik} = \dot{p}_{ik} + \dot{q}_{ik}, \qquad \dot{q}_{ik} = \frac{d}{dt}\frac{\partial R}{\partial\sigma^{ik}}, \qquad \dot{p}_{ik} = \frac{A}{n+1}\Gamma^{-d}\frac{\partial T^{n+1}}{\partial\sigma^{ik}}$$

then

$$\delta K_1 = \iiint\limits_V \left\{\frac{A}{n+1}\Gamma^{-d}\frac{\partial T^{n+1}}{\partial\sigma^{ik}} + \frac{d}{dt}\frac{\partial R}{\partial\sigma^{ik}}\right\}\delta\sigma^{ik}dV - \iiint\limits_V \dot{\mathbf{U}}\cdot\delta\mathbf{Q}dV - \iint\limits_S \dot{\mathbf{U}}\cdot\delta\mathbf{P}dS =$$
$$= \delta\iiint\limits_V \left\{\frac{A}{n+1}\Gamma^{-d}T^{n+1} + \frac{d}{dt}R\right\}dV - \delta\iiint\limits_V \mathbf{U}\cdot\mathbf{Q}dV - \delta\iint\limits_S \dot{\mathbf{U}}\cdot\mathbf{P}dS$$

Thus, $\delta K = \delta K_1$ and the difference $K - K_1$ is not dependent on the stress, but for the actual state $\delta K_1 = 0$. In fact, since the statical conditions are not violated

$$\delta\mathbf{Q} = -\nabla_i\delta\sigma^{ik}\mathbf{r}_k, \qquad \delta\mathbf{P} = -\delta\sigma^{ik}\mathbf{r}_k n_i$$

Hence

$$\delta K_1 = \iiint\limits_V \dot{\varepsilon}_{ik}\delta\sigma^{ik}dV + \iiint\limits_V \dot{\mathbf{U}}\cdot\nabla_i\delta\sigma^{ik}\mathbf{r}_k dV + \iint\limits_S \dot{\mathbf{U}}\cdot\delta\sigma^{ik}\mathbf{r}_k n_i dS =$$
$$= \iiint\limits_V \left\{ \dot{\varepsilon}_{ik} - \frac{1}{2}\,(\nabla_i \dot{U}_k + \nabla_k \dot{U}_i) \right\}\delta\sigma^{ik}dV$$

Here we use the Ostrogradskii-Gauss theorem. Thus, if the compatibility conditions for velocity of deformations are satisfied then $\delta K_1 = 0$. Conversely, from $\delta K_1 = 0$ follow the compatibility conditions of the velocities of deformation. Consequently, for the actual state $\delta K = 0$. Therefore, for thin plates and shells, with the assumption

$$\iiint\limits_V \dot{\mathbf{U}}\cdot\delta\mathbf{Q}dV + \iint\limits_S \dot{\mathbf{U}}\cdot\delta\mathbf{P}dS = 0$$

one obtains from (4.1) the variational equation

$$\delta K_* = \delta\iint\limits_{S_0}\int\limits_{-h}^{h}\left\{\frac{A}{n+1}\,\Gamma_*^{-d}T_*^{\,n+1} + \frac{dR_*}{dt}\right\}dz dS_0 = 0 \tag{4.6}$$

where

$$R_* = U_* + \int\limits_0^{\tau_*}\theta(\tau_*)\,\tau_* d\tau_*, \qquad U_* = U\,|_{\sigma^{33}=0}, \qquad \tau_*^{\,2} = \frac{1}{2}\,(\sigma^{\alpha\beta}\sigma_{\alpha\beta} - 3\sigma_*^{\,2}), \qquad 3\sigma_* = \sigma_{\alpha\cdot}^{\alpha} \tag{4.7}$$

The distribution of stresses for the state of unsteady creep can be written as

$$\sigma^{\alpha\beta} = \sigma_{(0)}^{\ \ \alpha\beta} + \lambda(t)\,(\sigma_{(c)}^{\ \ \alpha\beta} - \sigma_{(0)}^{\ \ \alpha\beta}) \tag{4.8}$$

as was suggested in [3]. Here $\sigma_{(0)}^{\ \ \alpha\beta}$ represents the initial distribution of the elasto-plastic stress and $\sigma_{(c)}^{\ \ \alpha\beta}$ represents the distribution of the stresses resulting from creep.

Since $\sigma_{(0)}^{\ \ \alpha\beta}$ and $\sigma_{(c)}^{\ \ \alpha\beta}$ are statically admissible, $\sigma^{\alpha\beta}$ is a statically admissible stress distribution. For $\lambda(t)$ we have the initial condition $\lambda(0) = 0$. Since (4.8) does not satisfy conditions (4.4), instead of Equation (4.6) one should take

$$\delta K_* = \iint\limits_{S_0}\int\limits_{-h}^{h}\left\{\frac{A}{2}\,\Gamma_*^{-d}T_*^{\,n-1}\,\frac{\partial T_*^{\,2}}{\partial\sigma^{\alpha\beta}} + \frac{d}{dt}\,\frac{\partial R_*}{\partial\sigma^{\alpha\beta}}\right\}\delta\sigma^{\alpha\beta}dz dS_0 = 0 \tag{4.9}$$

as the original equations.

Since

$$\delta\sigma^{\alpha\beta} = (\sigma_{(c)}^{\ \ \alpha\beta} - \sigma_{(0)}^{\ \ \alpha\beta})\,\delta\lambda \qquad \left(\frac{d}{dt}\quad \delta\sigma^{\alpha\beta} \neq 0\right)$$

hence

$$\iint_{S_0}\int_{-h}^{h}\left\{\frac{A}{2}\Gamma_*^{-d}T_*^{n-1}\frac{\partial T_*^2}{\partial\sigma^{\alpha\beta}}+\frac{d}{dt}\frac{\partial R_*}{\partial\sigma^{\alpha\beta}}\right\}(\sigma_{(c)}{}^{\alpha\beta}-\sigma_{(0)}{}^{\alpha\beta})dzdS_0=0 \tag{4.10}$$

This equation is used in order to obtain $\lambda(t)$ and can be written in the form

$$a(\lambda)\frac{d\lambda}{dt}+b(\lambda)=0 \tag{4.11}$$

The solution of this equation can be obtained using numerical methods (with the condition $\lambda(0) = 0$).

Let the initial deformations be within the range of elasticity. Then for a circular thin plate with a clamped boundary one obtains

$$U_3=\frac{r^4q}{64D}(1-\eta^2)^2, \qquad D=\frac{2Eh^3}{3(1-\nu^2)}$$

$$\sigma_{(0)}{}^{11}a_{11}=\frac{3r^2qz}{32h^3}[1+\nu-(3+\nu)\eta^2], \qquad \sigma_{(0)}{}^{22}a_{22}=\frac{3r^2qz}{32h^3}[1+\nu-(1+3\nu)\eta^2]$$

$$T_*{}^2=\frac{3}{2}(\sigma^{\alpha\beta}\sigma_{\alpha\beta}-3\sigma_*{}^2), \qquad R_*=\Pi_*=\frac{1}{2}A^{\alpha\beta\rho\gamma}\sigma_{\alpha\beta}\sigma_{\rho\gamma} \tag{4.12}$$

$$3\sigma_*=\sigma_{\alpha}^{\cdot\alpha}, \qquad q_{\alpha\beta}=A_{\alpha\beta\rho\gamma}\sigma^{\rho\gamma}=A_{\alpha\beta\cdot\cdot}^{\cdot\cdot\rho\gamma}\sigma_{\rho\gamma}$$

Consequently, from Equation (4.11) it follows that

$$a(\lambda)=\iint_{S_0}\int_{-h}^{h}A^{\alpha\beta\rho\gamma}(\sigma_{\alpha\beta}^{(0)}-\sigma_{\alpha\beta}^{(c)})(\sigma_{\rho\gamma}^{(0)}-\sigma_{\rho\gamma}^{(c)})\,dzdS_0$$

$$\begin{aligned} b(\lambda)=\iint_{S_0}\int_{-h}^{h}\Bigg\{&A\Gamma_*^{-\alpha}T_*^{n-1}\frac{3}{2}(\sigma^{\alpha\beta}-a^{\alpha\beta}\sigma_*)+\sigma_{\rho\gamma}\frac{dA^{\alpha\beta\rho\gamma}}{dt}+\\ &+A^{\alpha\beta\rho\gamma}\left[\frac{d\sigma_{\rho\gamma}^{(0)}}{dt}+\lambda(t)\left(\frac{d\sigma_{\rho\gamma}^{(c)}}{dt}-\frac{d\sigma_{\rho\gamma}^{(0)}}{dt}\right)\right]\Bigg\}(\sigma_{\alpha\beta}^{(c)}-\sigma_{\alpha\beta}^{(0)})\,dzdS_0 \end{aligned} \tag{4.13}$$

If the elastic constants are not dependent on the temperature then $dA^{\alpha\beta\rho\gamma}/dt = 0$, and for q = const we have $d\sigma_{(0)}^{\alpha\beta}/dt = 0$. Then

$$-b(\lambda)=\iint_{S_0}\int_{-h}^{h}\left\{A\Gamma_*^{-\alpha}T_*^{n-1}\frac{3}{2}(\sigma^{\alpha\beta}-\sigma_*a^{\alpha\beta})+\lambda(t)A^{\alpha\beta\rho\gamma}\frac{d\sigma_{\rho\gamma}^{(c)}}{dt}\right\}(\sigma_{\alpha\beta}^{(c)}-\sigma_{\alpha\beta}^{(0)})\,dzdS_0$$

Thus, we have succeeded in matching the distribution of instantaneous elasto-plastic stresses with creep stresses. We have obtained the stress distribution (4.8) continuous with respect to time.

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